NKT/KS/17/5199

Bachelor of Science (B.Sc.) Semester—VI (C.B.S.) Examination

M_{12}–DISCRETE MATHEMATICS AND ELEMENTARY NUMBER THEORY

Paper–2

(Mathematics)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. 1 to 4 have an alternative. Solve each question in full or its alternative in full.

UNIT–I

1. (A) Given that A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and a relation R on A is such that R = {(x, y)/ x + y = 10}. Show that R is neither reflexive nor transitive but it is symmetric. 6

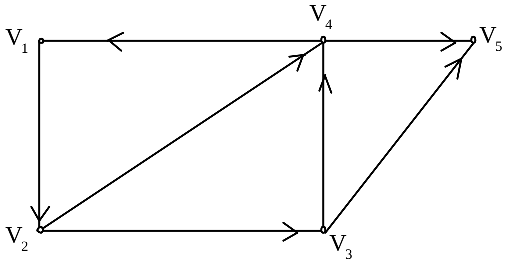
(B) Let (L, ≤) be a Lattice. For any a, b, c, ∈ L; if * and ⊕ are operations of meet and join, then prove that :

$$b \le c \Rightarrow \begin{cases} a * b \le a * c \\ a \oplus b \le a \oplus c \end{cases}$$ 6

OR

(C) Define a distributive lattice and show that every chain is a distributive lattice. 6

(D) Define : Indegree, Outdegree and Total degree of node. Find all the indegree and outdegree of the digraph. 6



UNIT–II

2. (A) Prove that if g is greatest common divisor of b and c, then there exist integers x_0 and y_0 such that $g = (b, c) = bx_0 + cy_0$. 6

(B) Find the value of x and y that satisfy the equation :

$243x + 198y = 9$ 6

OR

(C) Let p be an odd prime and (a, p) = 1. Then prove that

$a^{p-1} \equiv 1 \pmod{p}$. 6

(D) Evaluate :

(n, n + 1) and [n, n + 1]. 6

UNIT–III

3. (A) Define Legendre's Symbol. Prove that if p is an odd prime, then

$$\left(\frac{-1}{p}\right) = \begin{cases} 1 & \text{if} \quad p \equiv 1 \pmod 4 \\ -1 & \text{if} \quad p \equiv 3 \pmod 4 \end{cases}$$ 6

(B) Evaluate $\left(\frac{-42}{61}\right)$ and $\left(\frac{30}{71}\right)$. 6

OR

(C) Solve the congruence if it is solvable :

$x^2 \equiv 7 \pmod{31}$. 6

(D) Prove that the quadratic congruence :

$x^2 \equiv a \pmod{p}$ where p is odd positive prime integer with $a \neq kp$ for any integer k, has exactly two solutions, provided the solution exist. 6

UNIT–IV

4. (A) Prove that the positive primitive solution of $x^2 + y^2 = z^2$ with y even are given by $x = r^2 - s^2$, $y = 2rs$, $z = r^2 + s^2$

where r, s are arbitrary positive integers of opposite parity and $r > s$ and $(r, s) = 1$. 6

(B) Find all the primitive Pythagorean triples x, y, z such that $z - y = 1$. 6

OR

(C) Construct Farey Sequence F_6, given that $F_1 = \left\{\frac{0}{1}, \frac{1}{1}\right\}$. 6

(D) If $\frac{a}{b}$ and $\frac{a'}{b'}$ are two consecutive terms in F_n with $\frac{a}{b}$ to the left of $\frac{a'}{b'}$, then prove $a'b - ab' = 1$. 6

QUESTION–V

5. (A) Define an equivalence relation. Give an example of equivalence relation. 1½

(B) Define simple graph and multi-graph. 1½

(C) Prove that 4x $(n^2 + 2)$ for any integer n. 1½

(D) Define Reduced Residue System. Find reduced residue system for $m = 12$. 1½

(E) Prove that $\left(\frac{a^2}{p}\right)=1$. 1½

(F) Define greatest integer function [x]. Find [–3.5] and [12]. 1½

(G) Test the solvability of $4x + 8y = 2$. 1½

(H) Prove that terms in a Farey sequence are in monotonically increasing order. 1½

NKT/KS/17/5200

Bachelor of Science (B.Sc.) Semester—VI (C.B.S.) Examination

M_{12}–DIFFERENTIAL GEOMETRY

Paper–2

(Mathematics)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. 1 to 4 have an alternative. Solve each question in full or its alternative in full.

UNIT–I

1. (A) Prove that Darboux vector $\bar{d}$ is constant if K and τ are constant and the d has a fixed direction in $\frac{K}{\tau}$ is constant. 6

(B) Prove that the Frenet-Serret formulae :

(i) $\frac{d\bar{t}}{ds} = k\bar{n}$

(ii) $\frac{d\bar{n}}{ds} = \tau\,\bar{b} - K\bar{t}$

(iii) $\frac{d\bar{b}}{ds} = -\tau\,\bar{n}$

where K and τ are curvature and torsion of the space curve. 6

OR

(C) Show that the tangent to the locus of the centres of the osculating sphere passes through the centre of the osculating circle. 6

(D) Prove that the necessary and sufficient condition that a curve to be a helix is that ratio of curvature to torsion is constant at all points. 6

UNIT–II

2. (A) Find the involutes and evolutes of the circular helix :

$\bar{r} = (a \cos\theta, a \sin\theta, b\theta)$. 6

(B) Show that the curvature K_1 and torsion τ_1 of an involute $\tilde{c}$ of c are given by

$$K_1^2 = \frac{\tau^2 + K^2}{K^2 (c-s)^2}, \quad \tau_1 = \frac{K\tau' - K'\tau}{K(c-s)(K^2+\tau^2)}$$

where K and τ are the curvature and the torsion of the curve. 6

OR

(C) Find the principal curvatures at the origin of the paraboloid

$2\zeta = 5x^2 + 4xy + 2y^2$.

Also find the curvature of the section y = x. 6

(D) If $\dot{\alpha}\dot{\beta} - \dot{\beta}\dot{\alpha} \neq 0$, show that the line $x = a\zeta + \alpha$, $y = b\zeta + \beta$ generates a skew ruled surface and find its equation where a, b and α, β are functions of u. 6

UNIT–III

3. (A) Determine the unit normals and the fundamental forms of the surface

$\bar{r} = (a \cos u, a \sin u, bv)$. 6

(B) On the paraboloid $x^2 - y^2 = \zeta$, find the orthogonal trajectories of the section by the planes z = constant. 6

OR

(C) Obtain Gauss's formulae for $\bar{r}_{11}$, $\bar{r}_{12}$, $\bar{r}_{22}$, where $\bar{r}$ is the position vector of any point of a surface and suffixes 1 and 2 denotes differentiation with regard to u and *v* respectively. 6

(D) Prove that the Rodrigue's formula $K d\bar{r} + d\bar{N} = 0$

where K is the principle curvature. 6

UNIT–IV

4. (A) Define geodesic and further obtain the differential equation of the geodesic. 6

(B) Prove that the curves of the family $\frac{v^3}{u^2}$ = constant are geodesics on a surface with the metric.

$v^2\,du^2 - 2\,uv\,du\,dv + 2u^2dv^2,\ u > 0,\ v > 0$. 6

OR

(C) Find the Gaussian curvature at a point (u, v) of the anchor ring,

$r = ((b + a\cos u)\cos v, (b + a\cos u)\sin v, a\sin u),\ 0 < u,\ v < 2\pi$. 6

(D) Prove Bonnet's theorem for a curve on a surface that $w' + \tau = \tau_g$, where w is the normal angle and τ_g is the torsion of the geodesic tangent. 6

QUESTION–V

5. (A) Define Osculating Plane. 1½

(B) Define Fundamental plane at a point P whose position vector is $\bar{r}$ on the space curve $\bar{R}$. 1½

(C) Define Involute and Evolute. 1½

(D) Define Developable Surface. 1½

(E) Define a third fundamental form. 1½

(F) State Euler's theorem on normal curvature. 1½

(G) State Gauss-Bonnet theorem. 1½

(H) Define geodesic polar coordinates for the geodesic metric $ds^2 = du^2 + G(u, v)\,dv^2$. 1½

Bachelor of Science (B.Sc.) Semester—VI (C.B.S.) Examination

M_{12}–SPECIAL THEORY OF RELATIVITY

Paper–2

(Mathematics)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. 1 to 4 have an alternative. Solve each question in full or its alternative in full.

UNIT–I

1. (A) What is an inertial frame ? Show that in an inertial frame, a body not under influence of any forces, moves in a straight line with constant velocity. 6

(B) Prove that $\nabla^2 - \frac{1}{C^2}\frac{\partial^2}{\partial t^2}$ is invariant under special Lorentz transformations. 6

OR

(C) Explain Lorentz-Fitzgerald contraction idea. How was this idea used to account for negative result of the Michelson-Morley experiment ? 6

(D) Assume that the Lorentz transformations are the linear transformations of the form $x' = Ax + Bt$ and $t' = Dx + Et$, $E > 0$. Considering $x^2 - c^2 t^2 = x'^2 - c^2 t'^2$ and the motion of the origin of S, deduce the Lorentz transformations. 6

UNIT–II

2. (A) Obtain the transformation equations for the components of particle acceleration by using special Lorentz transformations. 6

(B) Explain the phenomenon of time dilation in special theory of relativity. 6

OR

(C) Obtain the transformation of Lorentz contraction factor $\left(1-\frac{u^2}{c^2}\right)^{\frac{1}{2}}$ in the two inertial frames of references. 6

(D) Prove that simultaneity is relative but not an absolute in special relativity. 6

UNIT–III

3. (A) Define symmetric and skew symmetric covariant tensors of order two. Show that any tensor of the second order may be expressed as the sum of a symmetric tensor and skew symmetric tensor. 6

(B) Define Kronckar Delta ∂^r_s. Show that ∂^r_s is a mixed tensor of order two. 6

OR

(C) Define space-like interval. Prove that there exists an inertial system S' in which the two events occur at one and the same time if the interval between two events is space-like. 6

(D) Define four tensor. Obtain the transformations of the components of a symmetrical four tensor T^{11} under the Lorentz transformations. 6

UNIT–IV

4. (A) If m is the relativistic mass of a particle moving with velocity u relative to an inertial frame s, then prove that :

$$m = \frac{m_o}{\left(1-\frac{u^2}{c^2}\right)^{\frac{1}{2}}}$$

where m_0 is the rest mass of the particle. 6

(B) Define four-velocity. Show that the four velocity, in component form, can be expressed as :

$$u^i = \left(\frac{\bar{u}}{c\sqrt{1-u^2/c^2}}, \frac{1}{\sqrt{1-u^2/c^2}}\right),$$

where $\bar{u} = (u_x, u_y, u_z)$. 6

OR

(C) Derive the wave equation for the propagation of the electric field strength $\overline{E}$ and the magnetic field strength $\overline{H}$ in free space with velocity of light. 6

(D) Obtain the transformation equations of the electromagnetic four-potential vector. 6

QUESTION–V

5. (A) Discuss the outcome of Michelson-Morley experiment regarding fringe shift and stationary ether. 1½

(B) Show that the circle $x'^2 + y'^2 = a^2$ in the frame of reference s' is measured to be an ellipse in s if s' moves with uniform velocity relative to s. 1½

(C) Derive Einstein's velocity addition law. 1½

(D) An electron is moving with a speed of 0.85 c in a direction opposite to that of a moving photon. Calculate relative velocity. 1½

(E) Define :

Free suffix and dummy suffix in a tensor quantity. 1½

(F) Show that two events which are separated by a time-like interval cannot occur simultaneously in any inertial system. 1½

(G) Show that the four velocity of a particle is a unit time-like vector. 1½

(H) Prove that the energy momentum tensor T^{ij} is symmetric. 1½